Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in
Further Pure Mathematics FP3
(6669/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2015
Publications Code UA041570

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - □ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1.	$2\left(1+\sinh^2 x\right)-3\sinh x=1$	Attempt to use $\cosh^2 x=1+\sinh^2 x$	M1
	$2\sinh^2 x-3\sinh x+1=0$	Correct 3 term quadratic. The "= 0" may be implied by their attempt to solve.	A1
	$(2\sinh x-1)(\sinh x-1)=0$	Attempts to solve their 3TQ = 0 leading to $\sinh x=$ ... (= 0 may be implied)	M1
	$\sinh x$ or $\dfrac{e^x-e^{-x}}{2}=\dfrac{1}{2}$ or 1	Both values correct	A1
	$x=\ln\frac{1}{2}\left(1+\sqrt{5}\right),\ \ln\left(1+\sqrt{2}\right)$	A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ or $\ln\left(1+\sqrt{2}\right)$ oe A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ and $\ln\left(1+\sqrt{2}\right)$ oe and **no other values**	A1, A1 **M1A1 on ePEN**
	Allow equivalent answers e.g. $\ln\left(\frac{1}{2}+\sqrt{\frac{5}{4}}\right),\ln\left(\frac{1}{2}+\sqrt{1+\frac{1}{4}}\right)$ and allow awrt 3SF accuracy e.g. $\ln 1.62$, $\ln 2.41$		
			(6)
			Total 6
	Alternative		
	$2\left(\dfrac{e^x+e^{-x}}{2}\right)^2-3\left(\dfrac{e^x-e^{-x}}{2}\right)=1$	Substitutes **correct** definitions for sinh$x$ and cosh$x$ in terms of exponentials	M1
	$e^{4x}-3e^{3x}+3e^x+1=0$	Correct quartic in e^x	A1
	$\left(e^{2x}-e^x-1\right)\left(e^{2x}-2e^x-1\right)=0\Rightarrow e^x=...$	Solves their quartic as far as e^x = ... For the **correct quartic** there must be a recognisable attempt to solve e.g. the product of two 3TQ's in e^x or if answers only are given, they must be correct (1.62, 2.41, and possibly (-0.618, -0.414)). For an **incorrect quartic** there must be a recognisable attempt to solve a quartic with at least 4 terms.	M1
	$e^x=\dfrac{1+\sqrt{5}}{2},\ \dfrac{2+\sqrt{8}}{2}$	Correct values for e^x. Allow $e^x=\dfrac{1\pm\sqrt{5}}{2},\ \dfrac{2\pm\sqrt{8}}{2}$ **but no incorrect values**. Allow awrt 1.62, 2.41	A1
	$x=\ln\frac{1}{2}\left(1+\sqrt{5}\right),\ \ln\left(1+\sqrt{2}\right)$	A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ or $\ln\left(1+\sqrt{2}\right)$ oe A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ and $\ln\left(1+\sqrt{2}\right)$ oe and **no other values**. allow awrt 3SF accuracy e.g. $\ln 1.62$, $\ln 2.41$	A1, A1 **M1A1 on ePEN**

Question Number	Scheme	Notes	Marks
2	$y=\cosh x \Rightarrow \frac{dy}{dx}=\sinh x$	Correct derivative	B1
	$\int\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}dx=\int\sqrt{1+\sinh^2 x}\,dx$	Uses the correct formula with their $\frac{dy}{dx}$	M1
	Alternative for first 2 marks: $y=\frac{e^x+e^{-x}}{2}\Rightarrow\frac{dy}{dx}=\frac{e^x-e^{-x}}{2}=$ B1 $\int\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}dx=\int\sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}dx=$ M1 Then apply the scheme		
	$=\int\cosh x\,dx$ or $\int\frac{e^x+e^{-x}}{2}dx$	Correct integral (Condone omission of dx)	A1
	$=[\sinh x]_1^{\ln 5}=\sinh(\ln 5)-\sinh(1)$	$\int\cosh x\,dx=\sinh x$ and correct use of the correct limits. **Dependent on the first method mark.**	dM1
	$=\frac{12}{5}-\frac{1}{2}\left(e-\frac{1}{e}\right)$	Or equivalent (must be in terms of e with no ln's) **Score when a correct answer is first seen and isw.**	A1cso
			(5)
	Some equivalent final answers: $\frac{12}{5}-\frac{e}{2}+\frac{e^{-1}}{2}$, $2.4-\frac{e-e^{-1}}{2}$, $\frac{12}{5}-\frac{e^2-1}{2e}$, $\frac{24e-5e^2+5}{10e}$		
	Special Case: $\frac{dy}{dx}=-\sinh x$ l**eads to a correct answer. This scores a maximum of 3/5 i.e. B0M1A1(recovery)dM1A0**		
			Total 5

Question Number	Scheme	Notes	Marks
3(a)	$\det(\mathbf{A}-\lambda\mathbf{I})=0$ or $\begin{vmatrix}2-\lambda & 1 & 0\\ 1 & 2-\lambda & 1\\ 0 & 1 & 2-\lambda\end{vmatrix}=0$	Either statement is sufficient. May also be implied by an attempt to form the characteristic equation	M1
	$(2-\lambda)((2-\lambda)^2-1)-(2-\lambda)=0$ or $(2-\lambda)\left[(2-\lambda)^2-2\right]=0$ $\left(\lambda^3-6\lambda^2+10\lambda-4=0\right)$	Recognisable attempt at characteristic equation – sign errors only.	M1
	$(2-\lambda)((\lambda^2-4\lambda+2))=0$		
	$\lambda=2,\ 2+\sqrt{2},\ 2-\sqrt{2}$ Allow awrt 3.41 and 0.586	B1: $\lambda=2$ **from any working** M1: Attempt to solve (usual rules) $\lambda^2-4\lambda+2=0$ A1: Obtains $2\pm\sqrt{2}$ oe e.g. $\frac{4\pm\sqrt{8}}{2}$	B1M1A1
			(5)
(b)	$\begin{pmatrix}2 & 1 & 0\\ 1 & 2 & 1\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=2\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ or $(2+\sqrt{2})\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ or $(2-\sqrt{2})\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ States or uses $\mathbf{A}\boldsymbol{x}=\lambda\boldsymbol{x}$ or $(\mathbf{A}-\lambda\mathbf{I})\boldsymbol{x}=\mathbf{0}$ for at least one of their eigenvalues		M1
	$\begin{pmatrix}1\\ 0\\ -1\end{pmatrix},\begin{pmatrix}1\\ \sqrt{2}\\ 1\end{pmatrix},\begin{pmatrix}1\\ -\sqrt{2}\\ 1\end{pmatrix}$ (any multiple of these)	A1: One correct eigenvector (allow awrt 1.41 for $\sqrt{2}$) A1: Two correct eigenvectors (allow awrt 1.41 for $\sqrt{2}$) A1: All eigenvectors correct (allow awrt 1.41 for $\sqrt{2}$)	A1 A1 A1 **No ft here**
	$\pm\begin{pmatrix}\frac{1}{\sqrt{2}}\\ 0\\ -\frac{1}{\sqrt{2}}\end{pmatrix},\pm\begin{pmatrix}\frac{1}{2}\\ \frac{1}{\sqrt{2}}\\ \frac{1}{2}\end{pmatrix},\pm\begin{pmatrix}\frac{1}{2}\\ -\frac{1}{\sqrt{2}}\\ \frac{1}{2}\end{pmatrix}$	All normalised and correct and exact. Allow equivalent forms e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\\ -1\end{pmatrix}$ (**Must be seen in (b)**)	A1 **No ft here**
			(5)
(c)	$\mathbf{P}=\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}\\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}2 & 0 & 0\\ 0 & 2+\sqrt{2} & 0\\ 0 & 0 & 2-\sqrt{2}\end{pmatrix}$	B1ft: One correct ft matrix. If awarding for **P** they must be using their normalised vectors B1ft: Both correct ft matrices and **P** consistent with **D**. The eigenvectors in **P** must be in the same order as the eigenvalues in **D**. **For both B marks it must be clear or implied which matrix is which. (NB: B0B1 is not possible)**	B1ft, B1ft
			(2)
			Total 12

Question Number	Scheme	Notes	Marks
4(a)	$x^2+2x-3=(x+1)^2-4$	$x^2+2x-3=(x\pm1)^2\pm\alpha\pm3,\ \alpha\neq0$	M1
	$\int\frac{1}{\sqrt{(x+1)^2-4}}\mathrm{d}x=\text{arcosh}\ \frac{(x+1)}{2}(+c)$ or $\ln\left\{(x+1)+\sqrt{(x+1)^2-4}\right\}$	M1: Use of arcosh (allow arccosh, $\cosh^{-1}$) Or uses $\ln\left\{x+\sqrt{x^2-a^2}\right\}$ A1: arcosh $\frac{(x+1)}{2}$ (+ c not required) Or $\ln\left\{(x+1)+\sqrt{(x+1)^2-4}\right\}$	M1 A1
			(3)
(b)	$S=\pi\int y^2\mathrm{d}x=\pi\int\left(\frac{1}{\sqrt{x^2+2x-3}}\right)^2\mathrm{d}x$	Use of $\int\pi y^2\mathrm{d}x$	M1
	$=\int\frac{1}{(x+1)^2-4}\mathrm{d}x=\left[\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)\right]$	M1: Use of $\ln\left(\frac{x\pm p}{x\pm q}\right)$ A1: $\int\frac{1}{(x+1)^2-4}\mathrm{d}x=\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)$	M1A1
	$==\frac{\pi}{4}\left(\ln\frac{1}{3}-\ln\frac{1}{5}\right)=\frac{\pi}{4}\ln\frac{5}{3}$	$\frac{\pi}{4}\ln\frac{5}{3}$	A1
	Special case: Uses $S=k\int y^2\mathrm{d}x$ scores a maximum M0M1A1A0		
			(4)
	NB: May use partial fractions in (b) for middle M1A1:		
	$\frac{1}{x^2+2x-3}\equiv\frac{1}{(x+3)(x-1)}\equiv\frac{1}{4}\left(\frac{1}{x-1}-\frac{1}{x+3}\right)$		
	$\int\frac{1}{(x+3)(x-1)}\mathrm{d}x=\left[\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)\right]$	M1: Use of $\ln\left(\frac{x\pm p}{x\pm q}\right)$ A1: $\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)$	M1A1
	Alternative for (b) by substitution:		
	$S=\pi\int y^2\mathrm{d}x=\pi\int\left(\frac{1}{\sqrt{x^2+2x-3}}\right)^2\mathrm{d}x$	Use of $\int\pi y^2\mathrm{d}x$	M1
	$u=x+1\Rightarrow\int\frac{1}{(x+1)^2-4}\mathrm{d}x=\int\frac{1}{u^2-4}\mathrm{d}u$		
	$\int\frac{1}{u^2-4}\mathrm{d}u=\left[\frac{1}{4}\ln\frac{u-2}{u+2}\right]$	M1: Use of $\ln\left(\frac{u\pm p}{u\pm q}\right)$ A1: $\frac{1}{4}\ln\frac{u-2}{u+2}$	M1A1
	$\pi\left[\frac{1}{4}\ln\frac{u-2}{u+2}\right]_3^4=\frac{\pi}{4}\left(\ln\frac{1}{3}-\ln\frac{1}{5}\right)=\frac{\pi}{4}\ln\frac{5}{3}$	$\frac{\pi}{4}\ln\frac{5}{3}$	A1
			Total 7

Question Number	Scheme	Notes	Marks
5(a)	$\mathbf{AB} = -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	Attempt $\pm(\mathbf{OB} - \mathbf{OA})$	M1
	$\mathbf{r} = \begin{pmatrix}1\\3\\2\end{pmatrix} + \lambda\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\left(\mathbf{r} - \begin{pmatrix}1\\3\\2\end{pmatrix}\right) \times \begin{pmatrix}-2\\-3\\-1\end{pmatrix} = 0$	Any correct vector form including the "**r** = " and the "= 0" "**r** = " can be "AB =" or "l = "etc. The direction can be any multiple of that shown.	A1
			(2)
(b)	$\frac{x-"1"}{"-2"} = \frac{y-"3"}{"-3"} = \frac{z-"2"}{"-1"}$ oe e.g. $\frac{x+1}{2} = \frac{y}{3} = \frac{z-1}{1}$	M1: Correct attempt at the Cartesian form using their position and direction A1: $\frac{x-1}{-2} = \frac{y-3}{-3} = \frac{z-2}{-1}$ oe	M1A1
			(2)
(c)	$\mathbf{AB} \times \mathbf{AC} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & -3 & -1\\1 & -2 & -2\end{vmatrix} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix} \times \begin{pmatrix}1\\-2\\-2\end{pmatrix} = \begin{pmatrix}4\\-5\\7\end{pmatrix}$ $(= \mathbf{AB} \times \mathbf{BC} = \mathbf{AC} \times \mathbf{BC})$	M1: Attempts vector product of 2 vectors **in the plane** e.g. $\mathbf{AB} \times \mathbf{BC}$ If there is no working, at least 2 components should be correct. A1: Any multiple of $4\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$	M1A1
	$\mathbf{r}.\begin{pmatrix}4\\-5\\7\end{pmatrix} = \begin{pmatrix}1\\3\\2\end{pmatrix}.\begin{pmatrix}4\\-5\\7\end{pmatrix}$ i.e. $\mathbf{r}.\begin{pmatrix}4\\-5\\7\end{pmatrix} = 3$ oe e.g. $\mathbf{r}.\begin{pmatrix}-4\\5\\-7\end{pmatrix} = -3$	dM1: Attempts scalar product using their normal vector and **a**, **b** or **c**. **Dependent on the previous M** A1: Correct equation (oe)	dM1A1
	See end of scheme for alternatives		
			(4)
(d)	$d = \frac{"3"}{\lvert"\mathbf{4i} - 5\mathbf{j} + \mathbf{7k}"\rvert} = \frac{3}{\sqrt{90}}$	M1: $d = \frac{\pm \text{their } p}{\lvert\text{their } \mathbf{n}\rvert}$, $p \neq 0$ A1: $\frac{3}{\sqrt{90}}$ oe e.g. $\frac{3}{3\sqrt{10}}, \frac{1}{\sqrt{10}}$, (awrt 0.316)	M1A1 **Note B1B1 on ePEN**
			(2)
	Alternative		
	$\lambda\begin{pmatrix}4\\-5\\7\end{pmatrix}.\begin{pmatrix}4\\-5\\7\end{pmatrix} = 3 \Rightarrow \lambda = \frac{1}{30} \Rightarrow d = \sqrt{\left(\frac{4}{30}\right)^2 + \left(\frac{5}{30}\right)^2 + \left(\frac{7}{30}\right)^2} = \frac{1}{\sqrt{10}}$		M1A1 **Note B1B1 on ePEN**
	M1: A correct method for finding "λ" and attempting the length of λ**n** A1: $\frac{3}{\sqrt{90}}$ oe e.g. $\frac{3}{3\sqrt{10}}, \frac{1}{\sqrt{10}}$, (awrt 0.316)		

Question Number	Scheme	Notes	Marks
6(a)	$y = x,\ y = -x$	Both required. Accept $y = \pm x$ and $x = \pm y$	B1
			(1)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cosh t}{\sinh t}$	Correct gradient	B1 **Note M1 on ePEN**
	$y - \sinh t = \dfrac{\cosh t}{\sinh t}(x - \cosh t)$	Correct straight line method. For $y = mx + c$ method, c must be found	M1
	$y\sinh t = x\cosh t - (\cosh^2 t - \sinh^2 t)$		
	$y\sinh t = x\cosh t - 1$*	Obtains the printed answer with at least one intermediate step.	A1* cso
			(3)
(c)	$y = x \Rightarrow x = \dfrac{1}{\cosh t - \sinh t},\ y = \dfrac{1}{\cosh t - \sinh t}$ $y = -x \Rightarrow x = \dfrac{1}{\cosh t + \sinh t},\ y = \dfrac{-1}{\cosh t + \sinh t}$	All four values correct. May be in exponential form e.g. $\left(\mathrm{e}^{t},\ \mathrm{e}^{t}\right)$ and $\left(\mathrm{e}^{-t},\ -\mathrm{e}^{-t}\right)$	B1
	$X = \dfrac{1}{2}\left(\dfrac{1}{\cosh t - \sinh t} + \dfrac{1}{\cosh t + \sinh t}\right)$ or $Y = \dfrac{1}{2}\left(\dfrac{1}{\cosh t - \sinh t} + \dfrac{-1}{\cosh t + \sinh t}\right)$	Correct attempt at X or Y. May be in exponential form e.g. $\left(\dfrac{\mathrm{e}^{t} + \mathrm{e}^{-t}}{2},\ \dfrac{\mathrm{e}^{t} - \mathrm{e}^{-t}}{2}\right)$	M1
	$X = \dfrac{1}{2}\left(\dfrac{\cosh t + \sinh t + \cosh t - \sinh t}{\cosh^2 t - \sinh^2 t}\right) = \cosh t$ $Y = \dfrac{1}{2}\left(\dfrac{\cosh t + \sinh t - \cosh t + \sinh t}{\cosh^2 t - \sinh^2 t}\right) = \sinh t$	Obtains $X = \cosh t$ and $Y = \sinh t$ May be shown using exponentials as above.	A1cso
			(3)
(d)	$A = \dfrac{1}{2}\sqrt{\dfrac{2}{(\cosh t - \sinh t)^2}}.\sqrt{\dfrac{2}{(\cosh t + \sinh t)^2}}$ Or e.g. $\dfrac{1}{2}\sqrt{2e^{2t}}\sqrt{2e^{-2t}}$	Correct triangle area method	M1
	$= \dfrac{1}{\cosh^2 t - \sinh^2 t} = 1$	Obtains an area of 1	A1
	So area is independent of t	Concludes independence of t having obtained a constant area. Conclusion must include the word independent (or not dependent) (but not e.g. just QED)	A1ft
			(3)
	Alternative area method: If A$\left(\frac{1}{\cosh t}, 0\right)$ is the intersection of QR with the x-axis Area OAR + Area OAQ = $\dfrac{1}{2} \times \dfrac{1}{\cosh t} \times \dfrac{1}{\cosh t - \sinh t} + \dfrac{1}{2} \times \dfrac{1}{\cosh t} \times \dfrac{1}{\cosh t + \sinh t}$ $= \dfrac{1}{2} \times \dfrac{1}{\cosh t} \times \left(\dfrac{1}{\cosh t - \sinh t} + \dfrac{1}{\cosh t + \sinh t}\right) = \dfrac{1}{2\cosh t} \times 2\cosh t = 1$		
			Total 10

Question Number	Scheme	Notes	Marks
7(a)	$I_n = \int \sin^{n-1} x \sin x \mathrm{d}x$	Split into $\sin^{n-1} x$ and $\sin x$	M1
	$I_n = \sin^{n-1} x(-\cos x) + \int (n-1)\sin^{n-2} x \cos^2 x\mathrm{d}x$	Integration by parts in the right direction (if the method is unclear or formula not quoted only allow sign errors) **Dependent on the first method mark.**	dM1
	$I_n = -\sin^{n-1} x\cos x + (n-1)(I_{n-2} - I_n)$	Obtains I_n correctly in terms of I_{n-2} and I_n	A1
	$I_n = -\sin^{n-1} x\cos x + (n-1)I_{n-2} - nI_n + I_n$		
	$I_n = \frac{1}{n}(-\sin^{n-1} x\cos x + (n-1)I_{n-2})$ *	**Printed answer** obtained with at least one intermediate step and no errors seen (condone the occasional x lost along the way but the final answer must be **exactly** as printed)	A1*
	Condone omission of "dx" throughout in both methods		**(4)**
	Alternative:		
	$= \int \sin^{n-2} x\left(1-\cos^2 x\right)\mathrm{d}x$	Splits into $\sin^{n-2} x$ and $\sin^2 x$ and uses $\sin^2 x = 1 - \cos^2 x$	M1
	$= I_{n-2} - \left\{\frac{\sin^{n-1} x\cos x}{n-1} + \int \frac{\sin^n x}{n-1}\mathrm{d}x\right\}$	Integration by parts in the right direction (if the method is unclear or formula not quoted only allow sign errors). **Dependent on the first method mark.**	dM1
	$= I_{n-2} - \frac{\sin^{n-1} x\cos x}{n-1} - \frac{1}{n-1}I_n$	Obtains I_n correctly in terms of I_{n-2} and I_n	A1
	$(n-1)I_n = (n-1)I_{n-2} - \sin^{n-1} x\cos x - I_n$		
	$I_n = \frac{1}{n}(-\sin^{n-1} x\cos x + (n-1)I_{n-2})$ *	Printed answer obtained with at least one intermediate step and no errors seen ((condone the occasional x lost along the way but the final answer must be **exactly** as printed)	A1*
(b)	$I_n = \frac{1}{n}(\left[-\sin^{n-1} x\cos x\right]_0^{\frac{\pi}{2}} + (n-1)I_{n-2})$	Use part (a) with limits	M1
	$I_n = \frac{n-1}{n}I_{n-2}$	Sight of the expression could score M1A1	A1
	n odd, $I_1 = \int_0^{\frac{\pi}{2}} \sin x\mathrm{d}x = \left[-\cos x\right]_0^{\frac{\pi}{2}} = 1$	An attempt at I_1 must be seen before any more marks are awarded	
	$I_n = \frac{(n-1)}{n}I_{n-2} = \frac{(n-1)}{n}\frac{(n-3)}{n-2}I_{n-4} = ...$	Attempts I_1 and at least 2 fractions in terms of n	M1
	$I_n = \frac{(n-1)(n-3)...6.4.2}{n(n-2)(n-4)...7.5.3}$ **	Cso. Note this may be awarded for 'extra' brackets top and bottom provided all previous marks are scored.	A1**
			(4)
(c)	$\int_0^{\frac{\pi}{2}} \sin^5 x\cos^2 x\mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin^5 x(1-\sin^2 x)\ \mathrm{d}x$	Uses $\cos^2 x = 1 - \sin^2 x$	M1
	$= I_5 - I_7 = \frac{4\times 2}{5\times 3} - \frac{6\times 4\times 2}{7\times 5\times 3}$	Correct numerical expression	A1
	$= \frac{8}{105}$	Cao (accept awrt 0.0761)	A1
	Correct answer only with no working would generally score no marks		**(3)**
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$b^2 = a^2(1-e^2) \Rightarrow e^2 = \dfrac{3}{4}$ or $e = \dfrac{\sqrt{3}}{2}$ NB $a = 2$, $b = 1$	M1: Uses a **correct** eccentricity formula to find a value for e or e^2 A1: $e^2 = \frac{3}{4}$ or $e = \frac{\sqrt{3}}{2}$ (allow $e = \pm\frac{\sqrt{3}}{2}$)	M1A1
	Foci: $(\pm ae, 0) \Rightarrow (\pm\sqrt{3}, 0)$	Both correct as coordinates	B1
	Directrices: $x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{4}{\sqrt{3}}$	Both directrices correct seen as equations. Accept un-simplified e.g. $x = \pm\frac{2}{\sqrt{3}/2}$	B1
			(4)
(b)	$PF_1 = ePN_1$ and $PF_2 = ePN_2$	Use of definition of ellipse for either PF_1 or PF_2	M1
	$PF_1 + PF_2 = e(PN_1 + PN_2) = eN_1N_2$	dM1: $(\text{their } e)\times 2\left(\text{their } \frac{4}{\sqrt{3}}\right)$ **Dependent on the previous method mark** A1: $\frac{\sqrt{3}}{2}\times\left(2\times\frac{4}{\sqrt{3}}\right)$	dM1A1
	$= 4$**	cso	A1**
			(4)
	(b) Alternative 1: Using $P(2\cos\theta, \sin\theta)$ **(Must be of this form)**		
	$PF_1 = \sqrt{(2\cos\theta-\sqrt{3})^2+\sin^2\theta}$ $PF_2 = \sqrt{(2\cos\theta+\sqrt{3})^2+\sin^2\theta}$	Correct use of Pythagoras for either PF_1 or PF_2	M1
	$PF_1 = \sqrt{(2-\sqrt{3}\cos\theta)^2}$ and $PF_2 = \sqrt{(2+\sqrt{3}\cos\theta)^2}$ dM1: Obtains both $PF_1{}^2 = \left(\sqrt{3}\cos\theta - \sqrt{p^2+1}\right)^2$ and $PF_2{}^2 = \left(\sqrt{3}\cos\theta + \sqrt{p^2+1}\right)^2$ where p is the x-coordinate of a focus. **Dependent on the previous method mark**		dM1
	$\lvert PF_1\rvert + \lvert PF_2\rvert = 2-\sqrt{3}\cos\theta+2+\sqrt{3}\cos\theta$	$2-\sqrt{3}\cos\theta+2+\sqrt{3}\cos\theta$. Note that if $\sqrt{3}\cos\theta-2$ is obtained correctly, it must become $2-\sqrt{3}\cos\theta$ to score any A marks	A1
	$= 4$**	cso	A1**
	(b) Alternative 2: Using $P\left(x, \sqrt{\dfrac{4-x^2}{4}}\right)$ **(Must be of this form)** or $P\left(\sqrt{4-4y^2}, y\right)$		
	$PF_1 = \sqrt{(x-\sqrt{3})^2+\dfrac{4-x^2}{4}}$ $PF_2 = \sqrt{(x+\sqrt{3})^2+\dfrac{4-x^2}{4}}$	Correct use of Pythagoras for either PF_1 or PF_2	M1
	$PF_1 = \sqrt{\left(2+\dfrac{\sqrt{3}}{2}x\right)^2}$ and $PF_2 = \sqrt{\left(2-\dfrac{\sqrt{3}}{2}x\right)^2}$ dM1: Obtains both $PF_1{}^2 = \left(\dfrac{\sqrt{3}}{2}x - \sqrt{p^2+1}\right)^2$ and $PF_2{}^2 = \left(\dfrac{\sqrt{3}}{2}x + \sqrt{p^2+1}\right)^2$ where p is the x-coordinate of the foci. **Dependent on the previous method mark**		dM1
	$\lvert PF_1\rvert + \lvert PF_2\rvert = 2-\dfrac{\sqrt{3}}{2}x+2+\dfrac{\sqrt{3}}{2}x$	$2-\dfrac{\sqrt{3}}{2}x+2+\dfrac{\sqrt{3}}{2}x$	A1
	$= 4$**	cso	A1**

(c)	Using chord as $y = mx + c$		
	$\frac{x^2}{4} + (mx+c)^2 = 1$	Substitutes the equation of a straight line with gradient m into the equation of the ellipse	M1
	$(1+4m^2)x^2 + 8mcx + 4(c^2 - 1) = 0$	Correct quadratic in x with terms collected	A1
	$x = \frac{1}{2}(\text{sum of roots}) = \frac{-4mc}{1+4m^2}$	Attempts $\frac{1}{2}(\text{sum of roots})$	M1
	$\Rightarrow c = -\frac{(1+4m^2)x}{4m}$	Correct expression for c in terms of m and x	A1
	So $y = mx - \frac{(1+4m^2)x}{4m}\left(= -\frac{1}{4m}x\right)$	ddM1: Substitutes back into $y = mx + c$ **Depends on both previous method marks** A1: Correct equation	ddM1A1
			(6)
	Or for last 3 marks:		
	$x = \frac{-4mc}{1+4m^2} \Rightarrow y = \frac{-4m^2c}{1+4m^2} + c\left(= \frac{c}{1+4m^2}\right)$	Correct y-coordinate in terms of m and c.	A1
	$y = -\frac{1}{4m}x$	ddM1: Obtains y in terms of x and m **Depends on both previous method marks** A1: Correct equation	ddM1A1
			Total 14

(c)	**Alternative:** Using factor formulae Let ends of the chord be $(2\cos\alpha, \sin\alpha)$ and $(2\cos\beta, \sin\beta)$ **(Must be of this form)**	
	$\left(\cos\alpha + \cos\beta, \frac{\sin\alpha + \sin\beta}{2}\right) = \left(2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right), \sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right)$ M1: Attempt mid-point **and** uses factor formulae A1: Correct mid-point	M1A1
	$m = \frac{\sin\beta - \sin\alpha}{2\cos\beta - 2\cos\alpha} = \frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{-4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}\left(= -\frac{1}{2}\cot\left(\frac{\alpha+\beta}{2}\right)\right)$ M1: Attempt gradient **and** uses factor formulae A1: Correct gradient	M1A1
	$y = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)}x$ and $m = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)} \Rightarrow y = -\frac{1}{4m}x$ ddM1: Uses the mid-point and gradient to establish an equation connecting y, m and x **Dependent on both previous method marks** A1: Correct equation	ddM1A1
		(6)

	Special Case: $x^2 + 4y^2 = 4 \Rightarrow 2x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$, *so* $m = -\frac{x}{4y}\left(y = -\frac{1}{4m}x\right)$ **Attempts like these that include further explanation should be sent to review.**	M1A1 **First 2 marks on ePEN**

Alternatives for 5(c)

	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\3\\2\end{pmatrix}+\lambda\begin{pmatrix}-2\\-3\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\-2\\-2\end{pmatrix}\Rightarrow 4x-5y+7z=3$	M1: Correctly forms the parametric equation and eliminates the parameters to obtain a cartesian equation A1: Correct cartesian equation	M1A1
	$4x-5y+7z=3\Rightarrow \mathbf{r}.\begin{pmatrix}4\\-5\\7\end{pmatrix}=3$	dM1: Converts their Cartesian equation into the form required. **Dependent on the previous M** A1: Correct equation (oe)	dM1A1

	$\begin{aligned}a+3b+2c&=d\\-a+c&=d\\2a+b&=d\end{aligned}\quad\Rightarrow a=\frac{4}{3}d, b=-\frac{5}{3}d, c=\frac{7}{3}d$	M1: Substitutes to obtain 3 equations in *a*, *b*, *c* and *d* and solves to obtain at least one of *a*, *b* or *c* in terms of *d* A1: Correct *a*, *b* and *c* in terms of *d*	M1A1
	$\frac{4}{3}x-\frac{5}{3}y+\frac{7}{3}z=1\Rightarrow \mathbf{r}.\frac{1}{3}\begin{pmatrix}4\\-5\\7\end{pmatrix}=1$	dM1: Uses their cartesian equation correctly to form a vector equation **Dependent on the previous M**	dM1A1

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom